Comparison of Several Spectral Estimation Methods for Application to ISAR Imaging

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SUMMARY

During the last decade, several spectral estimation techniques have been proposed for application to SAR/ISAR imaging. The present study attempts to shed light to a number of parametric spectral estimation methods, employed for ISAR imaging of aircraft targets. We focus on performance comparison with respect to 1-D and 2-D image resolution. Auto-regressive methods and MUSIC algorithm are examined and simulated, based on synthetic radar data, for both 1-D (range profiles) and 2-D (ISAR images) cases.

1 INTRODUCTION

In view of radar target recognition and classification, ISAR (inverse synthetic aperture radar) images shall be highly resolved (high-resolution imaging), so as to allow for decision making with respect to the target category and type. An important step in ISAR imaging is the implementation of high-resolution spectral estimation algorithms. Indeed, it is well-known that, under most circumstances, conventional Fourier transform processing of the two-dimensional raw radar data (2-D FFT) is not adequate to yield high-resolution radar aircraft images. Therefore, several spectral estimation techniques, both nonparametric and parametric, are applied in order to improve the resolution of both 1-D (i.e. range profiles) and 2-D (i.e. ISAR images) radar target images.

One-dimensional radar signatures of a target can be generated by applying spectral estimation techniques to the frequency domain data of the target, and it has been demonstrated that the resulting images are enhanced over those obtained via a Fourier transform-based technique. In bibliography, we come along extensions from 1-D spectral estimation-based radar signatures to two dimensions, where a Fourier transform of the angular domain data to the Doppler domain (cross-range) takes place. Hybrid methods for 2-D radar imaging employ an inverse Fourier transform on the frequency domain data, and then, a super-resolution technique is used to transform the angular domain data to the cross-range. Additionally, 1-D super-resolution spectral estimation methods can be applied to the frequency domain data and a superposition of the resulting 1-D signatures at different angles forms the desired 2-D radar image (known as backprojection technique). All these statements of bibliographic nature are more or less the history of 2-D spectral estimation-based radar imaging before the introduction of direct 2-D extensions of 1-D super-resolution spectral estimation methods [1]. Over the last decade, a great deal of research efforts has been devoted to this particular field and several ISAR imaging methodologies have been proposed [2] - [3].

In this paper, we compare and evaluate the performance of a number of parametric spectral estimation methods [4]. We firstly refer to the covariance and modified covariance methods, and the classical Burg method. Although these autoregressive (AR) model-based methods are more suitable for spectral estimation of signals with rational spectra, their computational simplicity makes them appealing for radar

Paper presented at the RTO SET Symposium on "Target Identification and Recognition Using RF Systems", held in Oslo, Norway, 11-13 October 2004, and published in RTO-MP-SET-080.

applications. Next, we elaborate on an eigenanalysis-based method, namely MUSIC (MUltiple SIgnal Classification), belonging to the very interesting class of super-resolution subspace methods. All the spectral estimation techniques mentioned in this paragraph are compared in the framework of simulated 1-D radar data, generated in accordance to the analysis of [5]. The resulting range profiles serve as valuable metrics of the achievable range resolution of each technique.

Spectral estimation methods for 2-D radar imaging are also examined in this work. Following the simulation setup described in [1], we generate 2-D simulated radar data in the frequency-angular domain, and then, we transform them to the spatial (2-D) frequency domain. Two-dimensional extensions of the classical AR [6] and MUSIC [1] methods are simulated, in order to perform 2-D spectral estimation and overcome the resolution limitations of the 2-D FFT. ISAR images are displayed via contour plots, indicating the advantages of each technique.

This paper is organized as follows. Section 2 refers to the format of the simulated 1-D and 2-D radar data on which the 1-D and 2-D spectral estimation techniques, described in Section 3, are applied. Section 3 is mainly focused on the 2-D methods, since 1-D spectral estimation methods are extensively described in [4]. Range profiles and ISAR images are the simulation results presented in Section 4, so as to compare the performance of the employed spectral estimation techniques. Conclusions and comments on the simulation results are derived and stated in Section 5, also including comments for future work.

2 SIMULATED RADAR DATA FORMAT

It is common knowledge that, in the high-frequency limit, a radar target can be considered as a collection of a finite number of scattering centers and scattering center interactions. The coherent scattered signal from such a radar target can be represented as the sum of complex scattered signals from each scattering center. As it is strongly stressed in [1], the assumption of a non-dispersive nature of the discrete scattering centers holds for most radar targets in the high-frequency limit, if the measurement bandwidth is narrow and the angular sector is small.

2.1 One-Dimensional Radar Data

Following the analysis of [5], we have simulated 1-D radar data (i.e. meaning that there is dependence on one coordinate), so as to compare the 1-D spectral estimation methods with respect to their range resolution. According to [5], the measured high-frequency radar cross-section (RCS) at frequency f_i can be represented by a sum of undamped exponentials as

$$y_i = \sum_{k=1}^{L} a_k \exp\left(-j\frac{4\pi}{c} f_i r_k\right) + n_i\ , \quad i = 1,2,...,N \qquad (1)$$

where r_k is the location of the *k*th scattering center, a_k is the corresponding amplitude, L is the number of scattering centers on the target, N is the number of frequency measurements (f_i), n_i is the measurement noise, and c denotes the speed of light.

2.2 Two-Dimensional Radar Data

In order to generate 2-D radar data in our simulations, we have followed the problem formulation of [1]. According to [1], the measured radar scattering signal from d scattering centers at frequency f_m ($m = 0,1,...,M-1$) and look angle θ_n ($n = 0,1,...,N-1$) is given as

$$x'(m,n)=\sum_{k=1}^{d} s_k \exp\left(j\frac{4\pi}{c} f_m \left[x_k \cos\theta_n - y_k \sin\theta_n\right]\right) + u(m,n) \qquad (2)$$

where s_k is the complex scattering intensity of the *k*th scattering center, x_k and y_k are the coordinates of the *k*th scattering center[1] on the rotation plane (*xy*-plane), $u(m,n)$ symbolizes the additive noise, usually assumed (as is the case in our simulations) to be white Gaussian with zero mean and variance σ^2.

In 2-D ISAR imaging, the frequency domain is transformed to the down-range and the angular domain to the cross-range. One can notice that the above 2-D data format contains two Fourier transform pairs not separable in terms of f and θ. Adopting the small angle approximation ($\cos\theta \approx 1$ and $\sin\theta \approx \theta$) is a way to obtain data consisting of two separable Fourier transform pairs, but the radar image generated using this approximation is unfocused. Interpolation of the frequency-angular domain data to Cartesian coordinates (rectangular grid of *M* x *N* points) with separable variables ($f^x = f\cos\theta$ and $f^y = f\sin\theta$) is the way to obtain a focused radar image. Rewriting the previous radar data equation we obtain

$$x(m,n)=\sum_{k=1}^{d} s_k \exp\left(j\frac{4\pi}{c} \left[f_m^x x_k - f_n^y y_k\right]\right) + u(m,n) \qquad (3)$$

In our simulations, we have performed this particular interpolation process described in [1], which is based on the following explanatory figure. Note that the "interpolated" data are available on a rectangular grid with equal increments in both the directions f^x and f^y, so that the fast Fourier transform (FFT) can be applied for the conventional radar image generation.

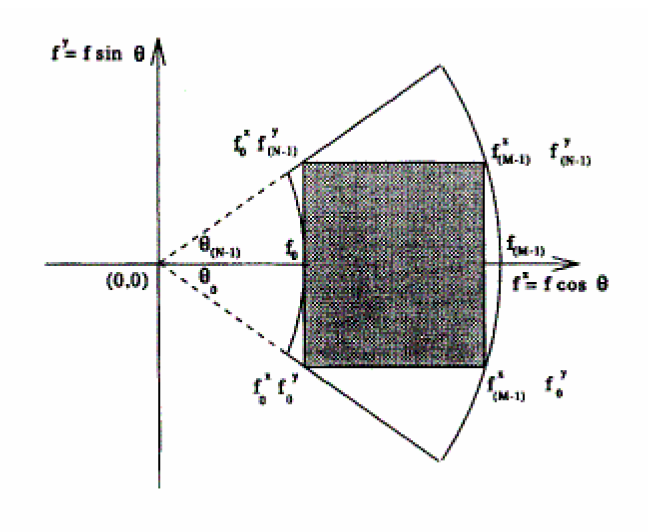

Figure 1. Frequency-angular domain of scattered radar data and the rectangular grid for the interpolation process (reproduced by [1])

It worths mentioning that the majority of high-resolution spectral estimation methods requires the computation of a correlation matrix, based on averaging over a number of snapshots of 1-D and 2-D radar data. Since only one snapshot is in most of the times available in radar applications (real-time processing),

[1] The *xy*-coordinates are relative to the center of rotation on the target.

there is need for decorrelating signals from various scattering centers. A special technique, known as spatial smoothing preprocessing (SSP) [7], and its modified version (MSSP – modified SSP) [8] have been proved sufficient to perform the required decorrelation of radar data. However, such preprocessing decreases the effective bandwidth and, consequently, results in reduced resolution. Because of this disadvantage, we have chosen not to simulate this preprocessing technique and allow some degree of correlation between the signals from different scattering centers. This is the reason for the presence of some redundant peaks in the resulting range profiles, not causing significant blurring in the final ISAR image.

3 SPECTRAL ESTIMATION METHODS FOR 1-D AND 2-D RADAR IMAGING

3.1 One-Dimensional Spectral Estimation Methods

3.1.1 AR Methods

For the generation of 1-D radar images (i.e. range profiles), we firstly employ three methods belonging to the class of autoregressive (AR) spectral estimation methods. These methods are called parametric, since they are based on the assumption of an autoregressive model structure for the signal being spectrally analyzed. Namely, the covariance, modified covariance and Burg methods are examined. We choose not to duplicate the details of each method, since they are widely covered in [4]. In general, all AR methods estimate the AR filter coefficients (or AR parameters) a_i ($i = 1,...,N$) and the average power σ^2 of the white noise, filtered by the autoregressive filter to produce a model of the examined signal. The resulting power spectral density (PSD) estimate is

$$P_{AR}(\omega) = \frac{\sigma^2}{|A(\omega)|^2} \quad \text{where} \quad A(\omega) = 1 + a_1 e^{-j\omega} + \ldots + a_N e^{-jN\omega} \qquad (4)$$

A common concept behind all three AR methods is the unified estimation of the correlation matrix and their main difference lies in the definition of the data matrix.

3.1.2 MUSIC Method

Another spectral estimation technique, also member of the wide class of parametric methods, simulated to obtain 1-D radar images is the MUSIC algorithm [4]. This method has been used for radar target recognition applications, providing super-resolved range profiles compared to those of the conventional IFFT for the same frequency bandwidth [5].
In the next two paragraphs, we shortly present the mathematical background of MUSIC for range profile generation. Rewriting equation (1) in vector notation we have

$$\mathbf{y} = \mathbf{Ea} + \mathbf{n} \qquad (5)$$

where $\mathbf{y} = [y_1, ..., y_N]^T$ is the data vector, $\mathbf{E} = [e(r_1), ..., e(r_L)]$ is a matrix with columns the N direction vectors of the form $e(r_k) = \left[\exp\left(-j\frac{4\pi}{c} f_1 r_k\right), ..., \exp\left(-j\frac{4\pi}{c} f_N r_k\right)\right]^T$, $\mathbf{a} = [a_1, ..., a_L]^T$ is the amplitude vector, and $\mathbf{n} = [n_1, ..., n_N]^T$ is the noise vector.

The first step of the MUSIC algorithm involves the computation of the correlation matrix of the radar scattering signal $\mathbf{y}$. The data correlation matrix is defined as

$$R_{yy} = \mathrm{E}[\mathbf{y}\,\mathbf{y}^{\mathrm{H}}] \qquad (6)$$

where we shall not forget that usually one data snapshot is available in radar applications, thus the ensemble average can be discarded from (6). Eigenanalysis of the correlation matrix is the second step of MUSIC, resulting in N eigenvectors, L of which correspond to the L maximum eigenvalues and constitute the signal subspace. The (N–L) eigenvectors, corresponding to the (N–L) minimum eigenvalues of the correlation matrix, form the noise subspace. The MUSIC algorithm takes advantage of the fact that the direction vector $e(r)$ is orthogonal to the noise eigenvectors u_i ($i = L+1, \ldots, N$), at each scattering center location $r = r_k$. The generation of the MUSIC pseudo-spectrum is the final step of the algorithm (estimated range profile) and is given by

$$P_{MUSIC}(r) = \frac{1}{\sum_{i=L+1}^{N} \left| u_i^H \, e(r) \right|^2} \qquad (7)$$

3.2 Two-Dimensional Spectral Estimation Methods

Two-dimensional ISAR images are obtained by applying 2-D spectral estimation techniques. In our simulations, we have developed 2-D MUSIC algorithm [1] and 2-D autoregressive spectral estimation (similar to the autocorrelation method in one dimension) [6].

3.2.1 MUSIC Method

Starting from the 2-D radar data format specified through equation (3), we present the basic theory of the 2-D MUSIC in the next paragraphs. Rewriting equation (3) in vector notation we have

$$\mathbf{x} = \mathbf{A}\mathbf{s} + \mathbf{u} \qquad (8)$$

where $\mathbf{x} = [x(0,0), x(1,0), \ldots, x(M-1,0), x(0,1), \ldots, x(M-1,N-1)]^T$ is the column-ordered data vector, $\mathbf{u} = [u(0,0), u(1,0), \ldots, u(M-1,0), u(0,1), \ldots, u(M-1,N-1)]^T$ is the column-ordered noise vector, $\mathbf{s} = [s_1, \ldots, s_d]^T$ is the scattering intensity vector, $\mathbf{A} = [a(x_1, y_1), \ldots, a(x_d, y_d)]$ is a matrix with columns the d direction vectors of the form

$$a(x_k, y_k) = \left[\exp\left(j\frac{4\pi}{c}\left[f_0^x\, x_k - f_0^y\, y_k\right]\right), \ldots, \exp\left(j\frac{4\pi}{c}\left[f_{M-1}^x\, x_k - f_0^y\, y_k\right]\right), \ldots \right.$$

$$\left. \ldots \exp\left(j\frac{4\pi}{c}\left[f_0^x\, x_k - f_1^y\, y_k\right]\right), \ldots, \exp\left(j\frac{4\pi}{c}\left[f_{M-1}^x\, x_k - f_{N-1}^y\, y_k\right]\right)\right]^T$$

Similarly to the 1-D case, the first step of the 2-D MUSIC algorithm is the computation of the correlation matrix, based on one snapshot of 2-D radar data. The data correlation matrix is defined as

$$R_{xx} = \mathrm{E}[\mathbf{x}\,\mathbf{x}^{\mathrm{H}}] \qquad (9)$$

Following the 1-D analysis, the next step of 2-D MUSIC involves the eigendecomposition of the correlation matrix. At this point, one should notice that the data vector $\mathbf{x}$ has length $M{\cdot}N$. As a result, the correlation matrix is square with dimension $M{\cdot}N$ and its eigendecomposition produces ($M{\cdot}N - d$) noise eigenvectors and d signal eigenvectors. All eigenvectors have the same length as the data vector. The eigenvectors of the noise subspace (corresponding to the ($M \cdot N - d$) minimum eigenvalues) form a matrix $\mathbf{E}_n$. The final step of the algorithm is the formation of the 2-D MUSIC pseudo-spectrum

$$P_{MUSIC}(x,y) = \frac{a(x,y)^H \, a(x,y)}{a(x,y)^H \, E_n \, E_n^H \, a(x,y)} \tag{10}$$

which indicates the positions of the scattering centers, identified as peaks of the pseudo-spectrum. Note that the scattering intensity information embodied in the 2-D MUSIC pseudo-spectrum (estimated ISAR image) is not very accurate. Therefore, amplitude estimation methods (such as least squares or weighted least squares) have to be applied separately from the 2-D MUSIC, if one desires more accurate estimation of the intensity of each scattering center.

3.2.2 AR Method

Two-dimensional AR spectral estimation can be implemented in a number of ways, starting from simple inversion of the data correlation matrix (direct method) and ending to iterative techniques, proposed to reduce the computational cost[2]. In the following paragraph, we briefly describe the direct method that is simpler than recursive solutions. In our simulations, we employ a small dataset and the computational burden of the direct method is not deterrent.

Typically, for the application of AR methods in two dimensions the data must be broken up into separate regions (i.e. sub-matrices of the original data matrix), each with distinct prediction error filter coefficients. The direct 2-D AR method for each region is summarized in the next lines. Firstly, the examined radar signal $x(n_1,n_2)$ is modeled as the output of a two-dimensional AR filter driven by white noise $e(n_1,n_2)$

$$x(n_1,n_2) = -\sum_{k=0}^{N_1-1}\sum_{l=0}^{N_2-1} a(k,l)\,x(n_1-k,n_2-l) + e(n_1,n_2) \tag{11}$$

To determine the AR filter coefficients from the available data, we define the prediction error filter (PEF), which is formed by stacking columns of the array of coefficients $a(i,j)$, as $\alpha = [1, a(0,1), ..., a(0,N_2-1), a(1,0), ..., a(1,N_2-1), a(2,0), ..., a(N_1-1,N_2-1)]^T$. The coefficients are chosen so as to minimize the mean square value of the prediction error, defined as

$$MSE = E\left[e^*(n_1,n_2)\,e(n_1,n_2)\right] = \alpha^H \, R_{xx} \, \alpha \tag{12}$$

where the data correlation matrix is $R_{xx} = \mathrm{E}[\mathbf{x}^* \, \mathbf{x}^T]$, with the column-ordered data vector given by $\mathbf{x} = [x(n_1,n_2), ..., x(n_1,n_2-N_2+1), x(n_1-1,n_2), ..., x(n_1-N_1+1,n_2-N_2+1)]^T$. With $\varepsilon_k(i)$ being a vector of length k whose ith element is unity, with all the other elements equal to zero, $\omega = [\omega_1, \omega_2]^T$ denoting the vector of angular frequencies (one for each dimension of the resultant spectral estimate), and the matrix E defined as

$$E = \left[\exp\left(-j\,\omega^T \begin{bmatrix}0\\0\end{bmatrix}\right), ..., \exp\left(-j\,\omega^T \begin{bmatrix}0\\N_2-1\end{bmatrix}\right), \exp\left(-j\,\omega^T \begin{bmatrix}1\\0\end{bmatrix}\right), ..., \exp\left(-j\,\omega^T \begin{bmatrix}N_1-1\\N_2-1\end{bmatrix}\right)\right]^T$$

[2] There is always a trade-off between computational cost and algorithmic complexity.

the 2-D AR estimate of the PSD of the 2-D radar signal is given by

$$P_{AR}(\omega_1,\omega_2)=\frac{\sigma^2}{\left|E^T\,\alpha\right|^2}=\frac{\varepsilon_{N_1N_2}(0)^T\,R_{xx}^{-1}\,\varepsilon_{N_1N_2}(0)}{\left|E^T\,R_{xx}^{-1}\,\varepsilon_{N_1N_2}(0)\right|^2} \qquad (13)$$

4 SIMULATION RESULTS

The simulated radar target is a simple wire model of an aircraft with nine scattering centers, drawn in the following figure. The maximum target extent is 15 m and it is initially nose-on to the radar. The position of the radar on the *xy*-plane is (0,0) and the center of rotation is set on the position (4530,0) on the target. The positions of the nine scattering centers are (4525,0), (4527,3.5), (4527,-3.5), (4530,0), (4530,7), (4530,-7), (4540,0), (4540,1.5), (4540,-1.5). The *xy*-coordinates in the next figure are with respect to the center of rotation.

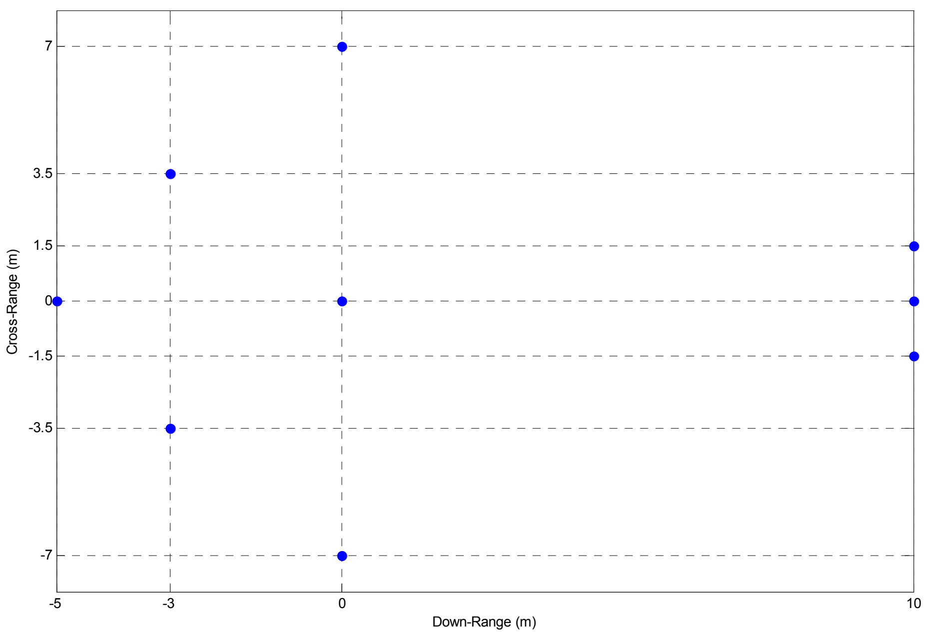

Figure 2. Simple aircraft model simulated as radar target

Taking into account the chosen parameters of the stepped frequency radar waveform (frequency step = 5 MHz, pulse repetition frequency = 10 KHz), the unambiguous down-range window is 30 m and the unambiguous maximum detection range is 15 km. Consequently, when plotting range profiles of the above target, we expect to identify scattering centers at down-range positions of 0 m (for down-range 4530 m), 10 m (for down-range 4540 m), 25 m (for down-range 4525 m) and 27 m (for down-range 4527 m). In our simulations, we have set the initial carrier frequency equal to 16.7 GHz, so as to imitate the frequency band of the imaging radar of the TIRA system[3].

As far as the Fourier processing for radar imaging is concerned, it is well-known that the corresponding down-range and cross-range resolution depend on the radar signal bandwidth and the aperture angle (i.e.

[3] TIRA stands for Tracking & Imaging RAdar. TIRA system (FGAN-FHR) is a radar system used to track and image air-borne targets in space and in atmosphere.

total look angle variation) respectively. Stepped frequency radar waveform is chosen in order to efficiently increase the signal bandwidth and improve the range resolution of FFT-based radar images. In our simulations, we do not employ windowed versions of the FFT, since there is always a trade-off between resolution and sidelobes' level (i.e. lower sidelobes are achieved at the cost of wider spectral peaks). Hanning and Hamming windows, for either one dimension or two dimensions, can satisfactorily reduce sidelobes.

The following two figures show range profiles generated by applying IFFT, covariance, modified covariance, Burg and MUSIC methods, for signal-to-noise ratio (SNR) of 25 dB and 10 dB. The stepped frequency waveform consists of 128 pulses (i.e. 128 frequencies), scanning a total signal bandwidth of 640 MHz. Note that the amplitude estimates for MUSIC-based range profile shall not be compared with those of the other four spectral estimates, since they are obtained by the MUSIC pseudo-spectrum.

The next three figures picture ISAR images generated by 2-D FFT, 2-D MUSIC and 2-D AR methods. The FFT-based image is obtained by simulating 512 frequencies and 200 look angles, so that its resolution is comparable to the one of the spectral estimation-based images. Note the dependence of resolution capabilities on the number of simulated frequency or angle steps. The other two images, based on 2-D spectral estimation, are obtained by simulating 16 frequencies and 31 look angles. Furthermore, considerable amount of white noise is added to the 2-D radar data (SNR = 25 dB). It worths noticing that the achievable resolution is sufficient for radar target recognition applications, even though very small datasets are simulated. The amplitude estimates are normalized to their maximum value (recall that the MUSIC-based amplitude estimates are derived from the 2-D pseudo-spectrum) and are quite similar. The extraneous peaks observed in MUSIC-based ISAR image are because of the correlation between data from different scattering centers. MSSP shall be employed in cases where the correlation causes severe image blurring.

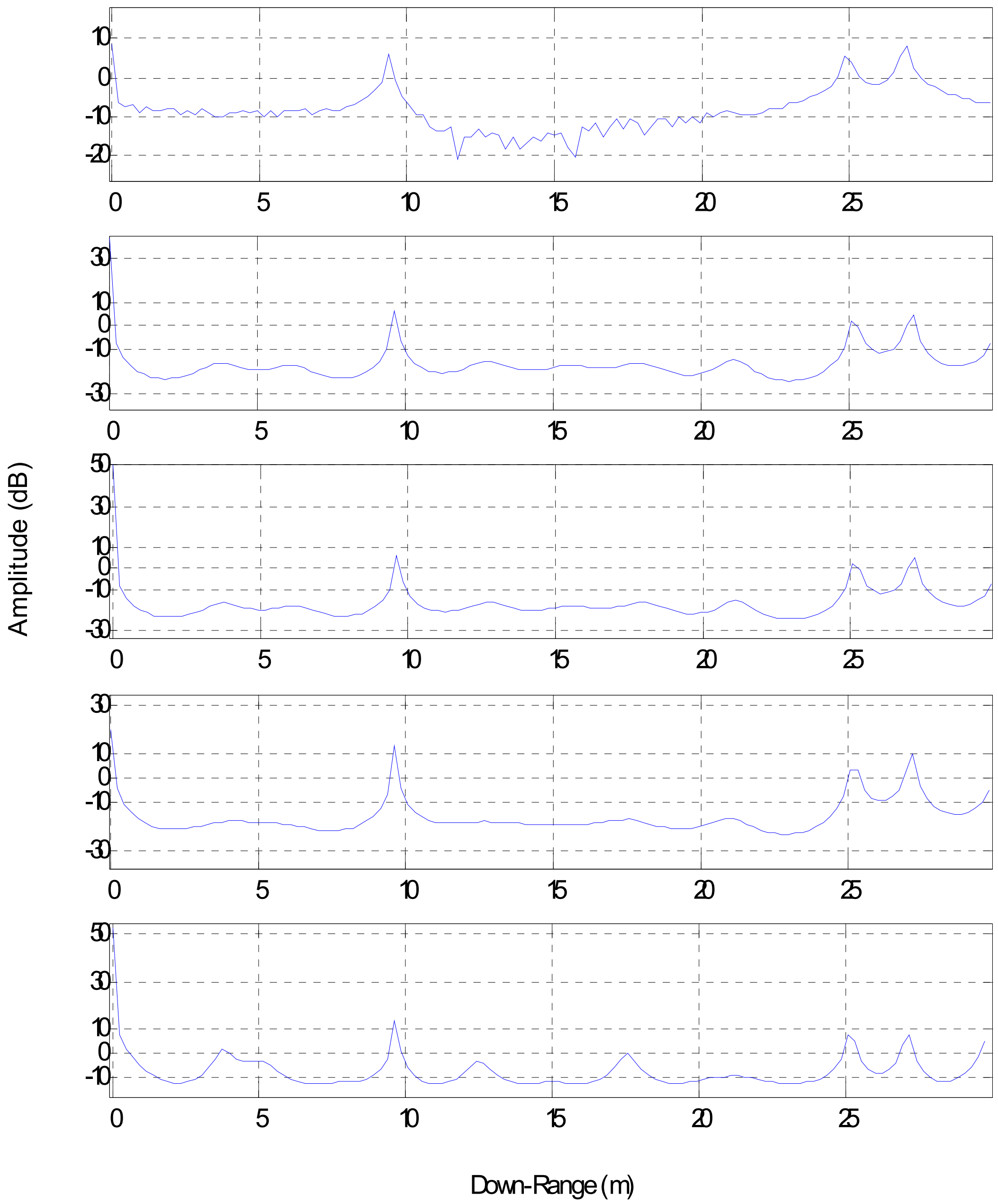

Figure 3. Range profiles generated respectively by applying IFFT, covariance, modified covariance, Burg and MUSIC methods, for SNR = 25 dB

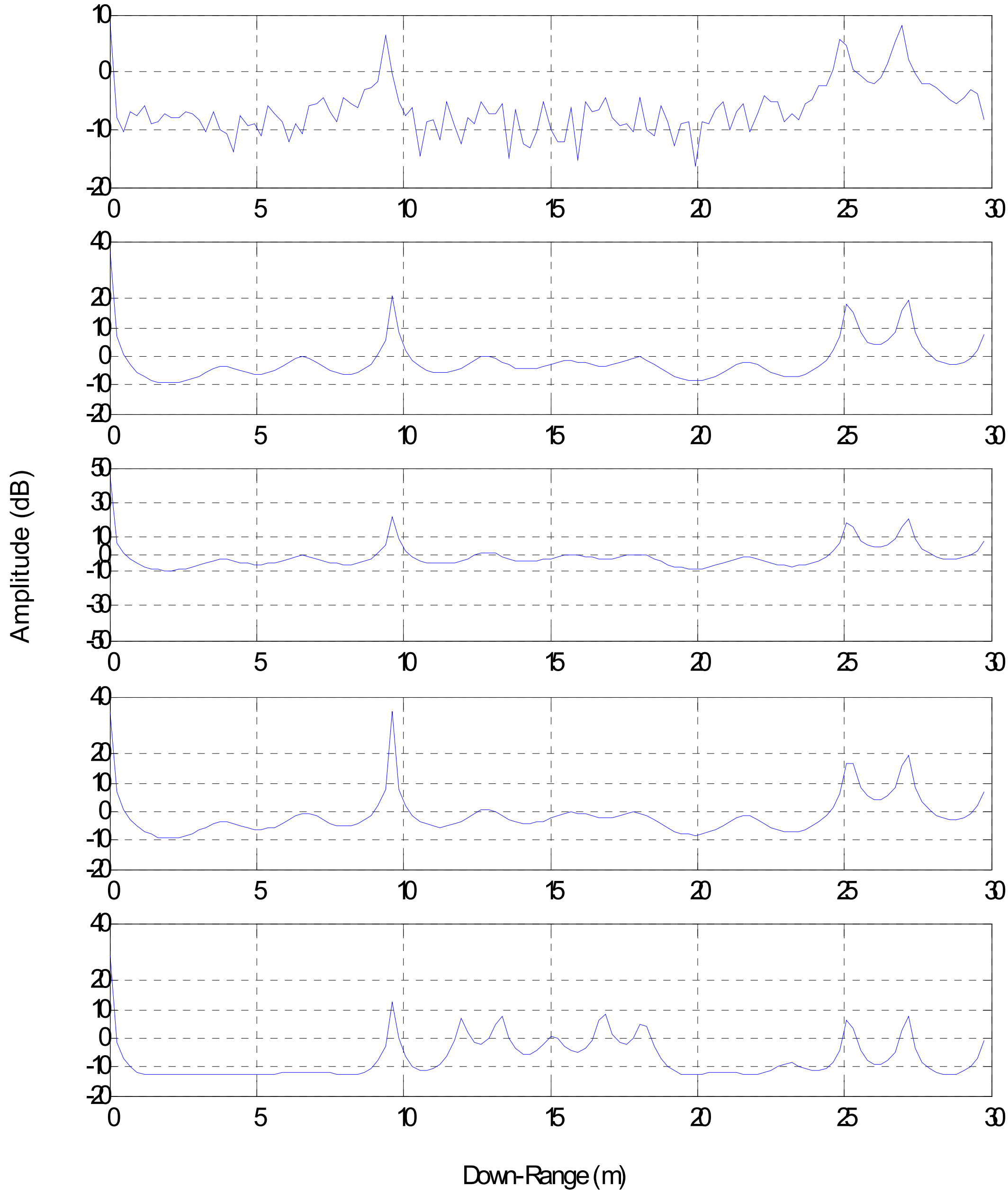

Figure 4. Range profiles generated respectively by applying IFFT, covariance, modified covariance, Burg and MUSIC methods, for SNR = 10 dB

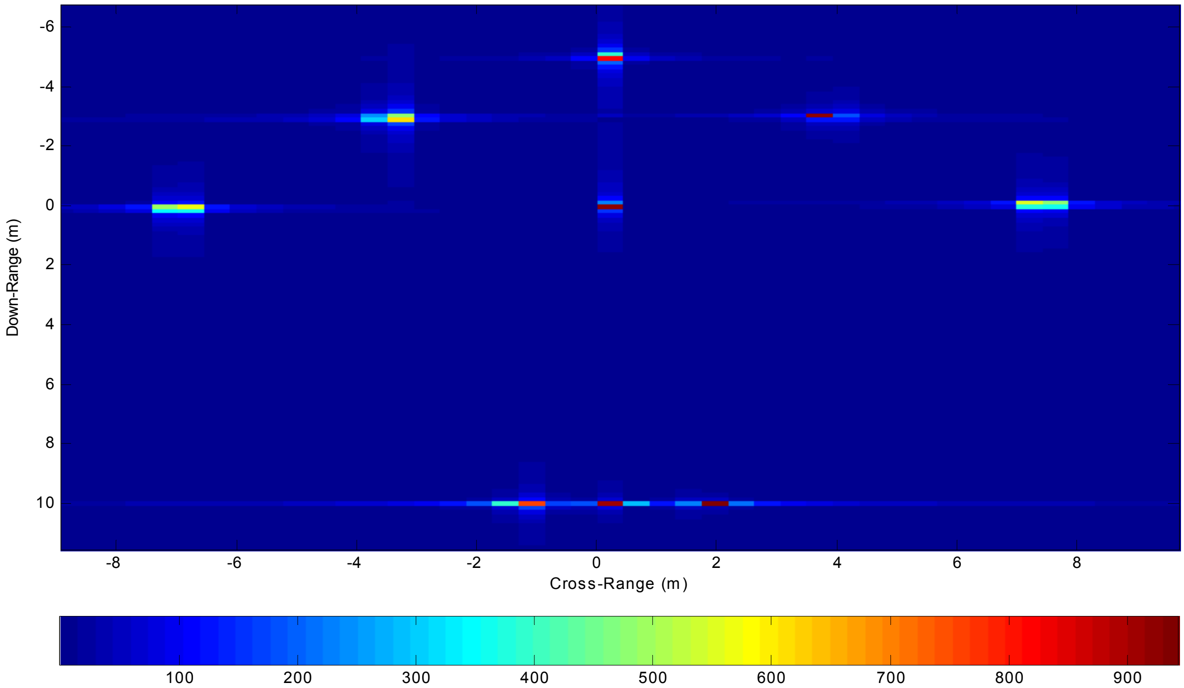

Figure 5. ISAR image generated by 2-D FFT (SNR = 40 dB)

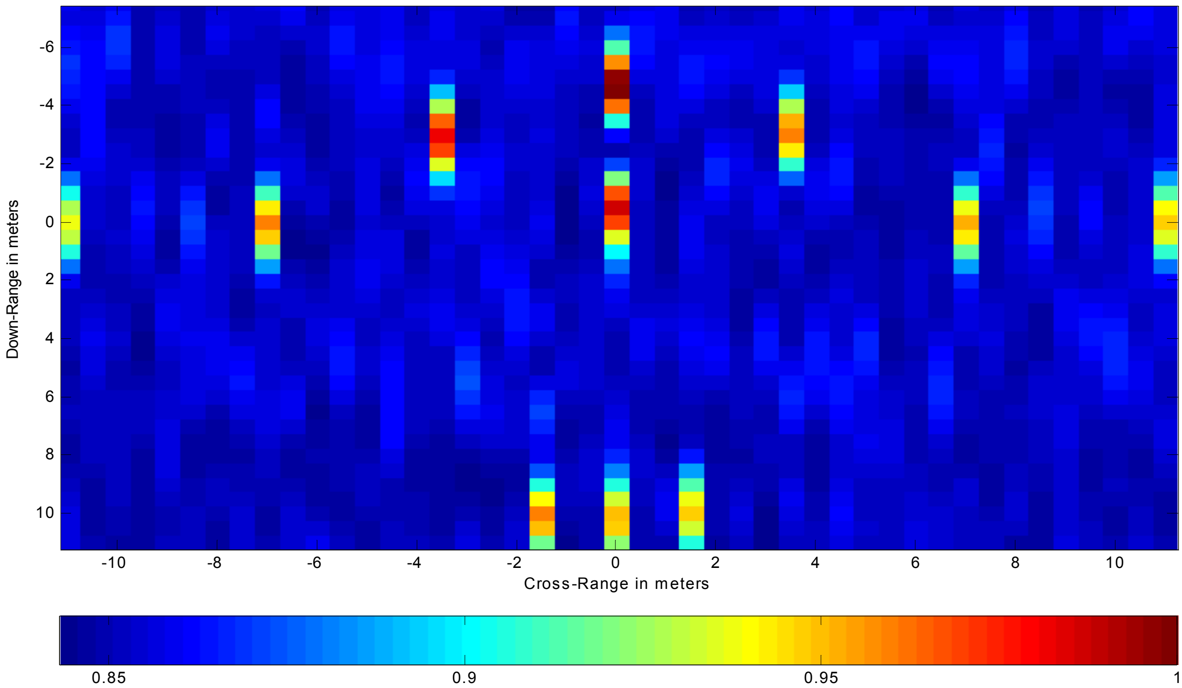

Figure 6. ISAR image generated by 2-D MUSIC (SNR = 25 dB)

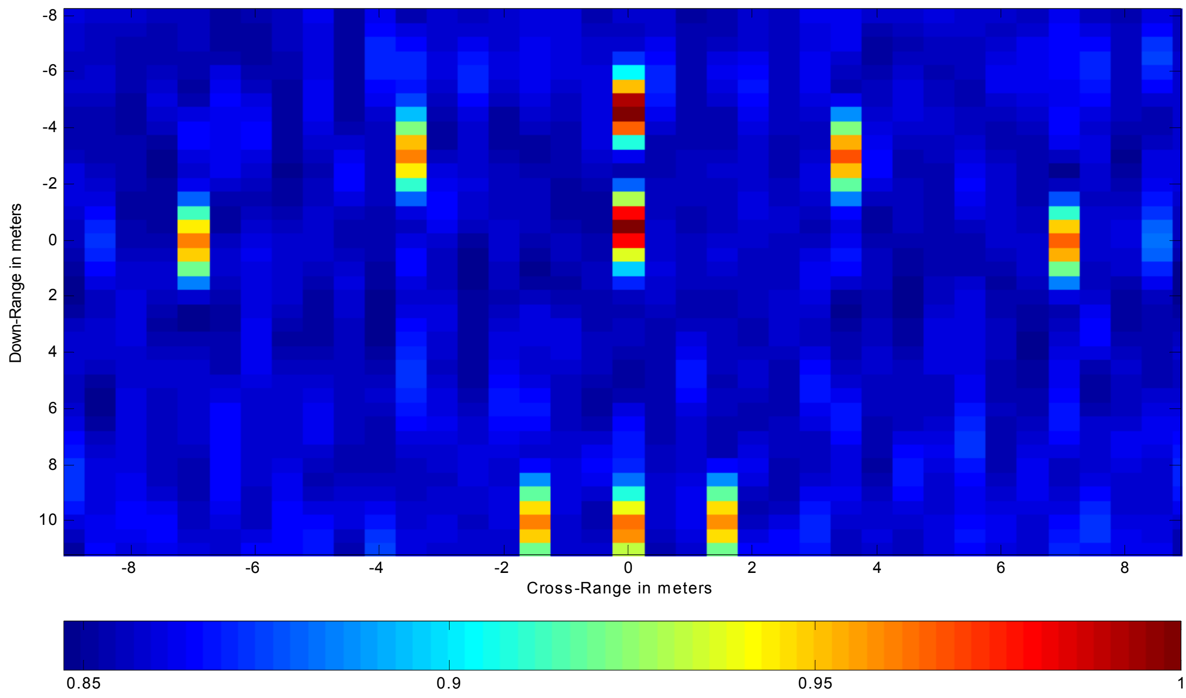

Figure 7. ISAR image generated by 2-D AR method (SNR = 25 dB)

5 CONCLUSIONS AND FUTURE WORK

Judging from the simulation results presented in Section 4 and those obtained through various simulation cases, we have reached the following conclusions with respect to the performance of spectral estimation-based radar imaging techniques:

(i) The spectral peaks of the IFFT-based range profiles are not as accurate (with respect to scatterers' ranges) as those of the spectral estimation-based range profiles. Moreover, higher sidelobes are evident in case of IFFT, compared to the smoother spectral estimates of AR methods (see Figure 3).

(ii) MUSIC-based range profiles, in case of moderate or high SNR, are more highly resolved than those of AR methods. Nonetheless, the problem of redundant spectral peaks shall be overcome by applying the MSSP technique, especially in case of low SNR values where the resulting range profile is severely worsened.

(iii) Burg method exhibits high-resolution capability for low SNR, whereas the sidelobes' level of IFFT-based range profiles increases as SNR decreases.

(iv) Spectral estimation-based ISAR images are characterized by satisfactorily high resolution, even for quite small datasets. On the other hand, the resolution of 2-D FFT-based images strongly depends on the number of frequencies (i.e. signal bandwidth) or look angles (i.e. aperture angle) simulated.

(v) Extraneous scattering centers, with low amplitude estimates, are identified by 2-D MUSIC pseudo-spectrum. In radar target recognition applications, MSSP may be necessary.

(vi) Two-dimensional AR spectral estimation and MUSIC result in quite similar ISAR image resolution. More complex target models would probably indicate the superiority of 2-D MUSIC.

The present study has motivated future research into three important topics:

(i) The computational cost of super-resolution spectral estimation techniques, especially for two-dimensional radar imaging, is the basic reason for choosing datasets as small as possible. The main advantage of FFT-based imaging is the computational speed. In order to lessen the computational burden, efficient techniques for computing the data correlation matrix shall be examined in future studies. The advantage of saving valuable memory space is of crucial importance for real-time radar signal processors.

(ii) Possible combinations between parametric (AR, MUSIC) and nonparametric high-resolution spectral estimation methods (Capon, APES) are under careful evaluation for two-dimensional radar imaging.

(iii) Above all, we are currently involved in processing real TIRA data provided by FGAN-FHR. Conventional FFT and super-resolution spectral estimation methods are to be applied on these measured data.

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